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Comparative study of fractional order series and parallel cascade control

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***Abstract*— This study presents a methodical design and comparison of the fractional-order parallel cascade controller (FOPCC) and fractional-order series cascade controller (FOSCC). There are two fractional order controllers in the planned both in FOPCC and FOSCC. For the primary loop and the secondary loop, separate fractional order controllers are used. In this study, we used fractional order PIDs to create a novel parallel cascade model and series cascade control model. To control realistic models with better output response, a novel parallel cascade control mechanism is required. The innovative structure offers improved solutions for first and second order systems in FOPCC with time delay. Fractional order differential equations can be used to express the fractional order controller. The theoretical foundation for deriving the controllers and analyzing the equations is provided by a number of fractional calculus laws. Using Smith Predictor in primary and secondary loops improves simulation results. Robustness, stability factors, and performance criteria through time response characteristics analysis have all undergone thorough evaluations. The ability to deliver a stable reaction to any process disruption is superior to the traditional series cascade controller compare to parallel cascade control. Examples are given to show how the recommended parallel cascade structure may be employed and how it performs better than the conventional series cascade control.**

Keywords*— Parallel Cascade Controller; Frictional Order cascade control; Smith predictor, Series Cascade Controller.

I. INTRODUCTION

Two controllers are used in cascading control, with the first controller supplying the second controller's set point. The feedback loop for one controller nests inside the other controller in a cascade controller. Power and temperature controllers, as well as sensors, are used to control the industrial heating process. The parts cooperate well to finish the thermal cycling of a procedure or product. When multiple sensors are available to monitor conditions in a regulated process, a cascade control system often outperforms a traditional single measurement controller. A cascade controller is an example of a steam fed water heater. The cascade controller may have disadvantages as well. Compared to a single measurement controller, it is more sophisticated. Cascade controller tuning is thus more difficult. The basic cascade control tuning strategy is to tune the primary controller first, then the secondary controller. Primary and secondary processes are connected in parallel rather than in series when using parallel cascade control. One example of parallel cascade control is the temperature control of sub-cooled relax by cascade control of exit cooling water temperature and reflux temperature via parallel transfer function. Another illustration of parallel cascade control is the parallel process transfer function-based overhead composition control of distillation column. In the chemical processing sector, a parallel cascade controller's primary function is to improve the dynamic performance of the control system under disturbance. The closed loop response is improved in series cascade control compared to unity feedback by using a middle sensor and controller to reject the disturbance before the controlled variable deviates from the set point.

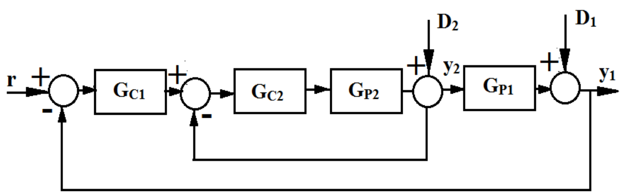

Fig. 1. Series cascade control

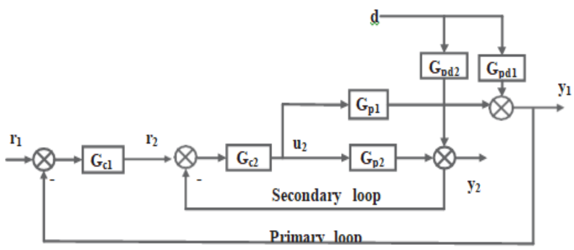

Fig. 2. Parallel cascade control

Figure-1, shows the basic block diagram of series cascade control. Here , G_{P1} represent the primary process, G_{P2} represent the secondary process, G_{c1} represent the primary controller, G_{c2} represent the secondary controller is the reference input, y_1 is the control output, D_1, D_2 represent disturbance of primary and secondary loops respectively. Figure-2 shows block diagram of PCCS in which the manipulated variable (u_2) and disturbance (d) simultaneously affected primary output (y_1) and secondary output (y_2), G_{P1} and G_{P2} are the transfer functions of primary and secondary process models. G_{c1} , G_{c2} are denotes the primary controller and secondary controller respectively. r_1 is the setpoint of primary loop and. r_2 is the set point of secondary loop. G_{Pd1} and G_{Pd2} represent the transfer function of disturbances entering the primary and secondary process output. The first person to apply a parallel cascade control system (PCCS) was Luyben [1]. Two real-world examples of parallel cascade control are temperature control of the subcooled reflux and overhead composition control of the distillation column. The best performance of PCCS was examined by Shen and Yu [2]; in this work, a decoupling of primary and secondary loop activities was seen since the real load disturbance differed from the expected one. Pottmann et al. proposed a PCCS for regulating arterial blood pressure in a biological system based on the H2optimal control theory [3]. Lee et al. updated the parallel cascade control structure by adding set point filters to the primary and secondary loops. A PCCS structure that increased output response and uncoupled primary and secondary loop control operations [4]. According to Nandong and Zang's report [5], the multi scale control method significantly improves the performance and durability of closed loop systems. two additional controllers, a modified PCCS, and a Pradhan and Majhi reported a set point filter for integrated and stable process models [6]. Pradhan and Majhi have released an enhanced PCCS for both stable and unstable process models. In this work, the lead-lag filter and PID in series served as the primary controller, and the IMC technique was used to build the secondary controller. Padhan and Majhi have described a PCCS for a class of stable, unstable, and integrating process models that ask for the tuning of two PID Controllers and a set point filter. The set point filter in the aforementioned studies was built using the integral squared error (ISE) performance criterion, whereas the PID controllers used the loop shaping technique. PCCS was altered to account for integrating, unstable, and stable process models. The secondary authors in the mentioned works. A fractional order parallel cascade control structure is introduced in the works of the present. Over series cascade control, fractional order parallel cascade control (FOPCC) has a number of benefits. (a) Better performance: Because FOPCC enables more accurate tweaking of the controller settings, it can perform better than conventional cascade control. Higher frequency gains adjustments are possible with fractional order controllers, which can lead to improved disturbance rejection and quicker response times. (b) Robustness: Compared to series cascade control, FOPCC is more resilient to system disruptions and uncertainty. This is so that any changes in the dynamics of the system can be accounted for using the additional degrees of freedom that fractional order controllers have. (c) Lessened sensitivity to noise: The control system's sensitivity to measurement noise can be lessened via FOPCC. By efficiently removing high-frequency noise, fractional order controllers can keep the low-frequency signals that are helpful for information while maintaining the high-frequency noise. (d) Lower energy consumption: The FOPCC can also lower the control system's energy usage. With their extra degrees of freedom, fractional order controllers can improve the control actions and consume less energy as a result. The suggested control strategy has the following advantages: (i) It is more resilient and performs better in closed loops than the existing methods. (ii) Several popular process models, such as stable, unstable, and integrating primary processes, can be implemented using the provided approach. Without using a nonlinear objective function, the tuning rule of the FOPCC Controller for primary processes can be produced directly. Fractional calculus is being employed to model linear and nonlinear fractional order processes in control systems. A fractional-order dynamic system, on the other hand, is highly helpful in expressing a variety of stable or unstable physical systems, improving versatility while requiring less computer power. Several straightforward techniques can be used by control engineers to implement fractional-order systems [33]. In generalization of the fractional order PID Controller, $PI^{\delta}D^{\mu}$ is including two extra parameters of the PID controller, as the fractional-order integrator (δ) and fractional- order differentiator (μ).The novelty of this work lies in the fact that fractional order parallel cascade control gives the better output response and time response specification, error rejection compare to fractional order series cascade control.

II.THEORY

2.1 Fractional Calculus

Fundamentally, differential and integral calculus is a specific case of fractional calculus. There are many definitions of fractional calculus in the literature, but the Riemann-Liouville formulation [7] is the most popular fractional order and is best suited for control fields.

$$aD_t^{v}g(t) = \frac{1}{\tau(n-v)}\frac{d^n}{d^n}\int_a^t \frac{g(\tau)}{(t-\tau)^{v-n+1}}\mathrm{d}t,\ (n-1)<v<n \quad (1)$$

In this case, τ(.) is Euler's gamma function; D is a functional operator; t and an are the upper and lower limits, respectively; v is the fractional order that indicates integration for v<0 and derivative for v>0.Equation no. (2) indicates the use of Laplace transformation to equations (1), where the order of v is selected such that 0<v<1.

$$L\left\{aD_t^{\pm v}g(t)\right\} = S^{\pm v}F(s) \quad (2)$$

All the initial conditions, in this case, are zero as the integer order is equal to one.

2.2. Fractional Linear Model

A linear time invariant (LTI) system with a single- input x(t), single output y(t), (SISO) the fractional- order differential equation (FODE) can be mathematically expressed as: $\sum_{j=0}^{m} b_i\, {}_0D^{\gamma j} y(t) = \sum_{i=0}^{n} a_i\, {}_0D^{\delta i} x(t)$

(3)

By applying the Laplace transform of equation (2) and (4) we get the generalized transfer function of fractional order system which is describe by the equation (5)

$$T(S) = \frac{y(S)}{X(S)} = \frac{b_{m}S^{\beta m} + b_{m-1}S^{\beta m-1} + \ldots\ldots\ldots\ldots b_0\beta^0}{a_{n}S^{\alpha n} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots a_0 S^{\alpha 0}} \quad (4)$$

2.3. The FOPID Controller in the Frequency Domain

The general dynamic system, the FOPID controller is of the form:

$$u(t) = k_p e(t) + k_i D^{-\lambda} e(t) + k_d D^{\mu} e(t) \quad (\lambda > 0)$$
(5)

The tuning parameters of FOPID have 5 degrees of freedom k_i, k_p, k_d, λ, and μ. After obtaining the values of k_p, k_i, and k_d by applying Ziegler-Nichols tuning rules, we have to set optimal values of λ and μ. This procedure can be followed by both taking different values of λ and μ and coming to the conclusion of the best fit values, or applying any optimization tuning rule for getting those values. Where k_p, k_i, k_d, are represent the proportional gain, integral time constant and derivative time constant respectively and λ, μ are the fractional order of the integration of integral and derivative terms respectively. By Appling the laplace transform of equation (5), Equation (6) can be represented in the form of transfer function.

$$G_C(s) = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu} \quad (6)$$

In equation (7), if the value of $\lambda = 1, \mu = 1$ It turns into a typical PID controller. Thus, in comparison to a normal controller, the FOPID controller has two extra tuning parameters, λ and μ. Fractional order provides several advantages, but the tuning process can be challenging. In order to determine the analytical tuning rules of the FOPID, a new design technique in the frequency domain was provided in this study [8]. The FOPID controller are converted in to frequency domain by substituting $s = j\varpi$ in the equation (6)

$$G_c(j\varpi) = k_p + \frac{k_i}{j\varpi^{\lambda}} + k_d j\, \varpi^{\mu} \quad (7)$$

The fractional power in Equation (7) can be representing by $j\varpi^{\lambda} = \varpi^{\lambda} j^{\lambda} = \varpi^{\lambda}\left[e^{j\left(\frac{\pi\lambda}{2} + 2n\pi\lambda\right)}\right]$ (8)

where n=0, $\pm\frac{1}{\lambda}$, $\pm\frac{2}{\lambda}$ ……..$\pm\frac{m}{\lambda}$. Finally, it can be approximate by the equations

$$j\varpi^{\lambda} = \varpi^{\lambda}\left[\cos\varphi + j\sin\varphi\right] \; \varphi = \frac{\pi\lambda}{2} \quad (9)$$

The complicated equation is transformed into the FOPID controller in the frequency domain by substituting (7) and (9)

$$G_c = \left(k_p + \frac{k_i \cos\varphi_i}{\varpi^{\lambda}} + \varpi^{\mu}\, k_d \cos\varphi\right) - j\left(\frac{k_i \sin\varphi_i}{\varpi^{\lambda}} + \varpi^{\mu}\, k_d \sin\varphi\right) \quad (10)$$

2.4. Sensitivity value for FOPID Controller

The maximum sensitivity of a single input single output system can be defined by

$$\psi_s = max\left|s(j\varpi)\right| \quad (11)$$

where s= (1+ξ), and ξ is an open- loop transfer function of the system. Ψ_s is the inverse of the shortest distance from the Nyquist curve of the open-loop transfer function to the critical point (-1, j0). It is seen that the higher value of Ψ_s give less robust for modelling uncertainties. The value Ψs needs to be close to 1 in order to improve system performance, robust stability, and both. Consequently, for integer-order control systems, the typical range of Ψs is selected between 1.4 and 2 [8,9]. This article considers the tuning value for fractional-order control systems in the context of an unstable primary and secondary process.

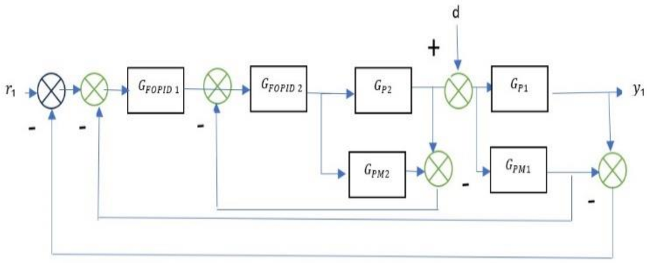

Fig. 3. Proposed fractional order Series cascade control

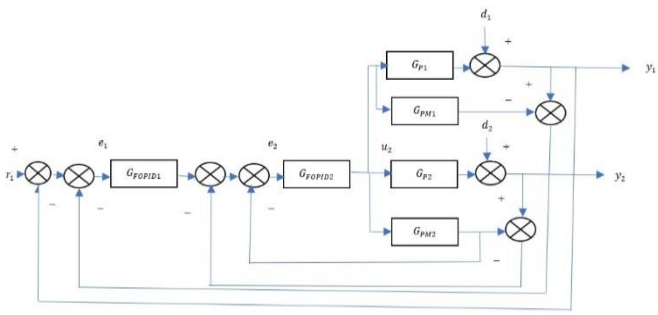

Fig. 4. Proposed fractional order parallel cascade control

2.5.1 Controller Design of FOSCC

Two separate fractional order controllers were employed in the series fractional order cascade control loop. Fractional-order series cascade controllers, which combine fractional-order controllers in a cascade structure to govern a system, require a number of design processes. Fractional-order controllers have better flexibility for handling complex and non-linear systems when compared to conventional integer-order controllers. First, fractional-order differential equations are used to create system models. In this suggested series of cascade control loops, the primary control loop uses a fractional-order PID (Proportional-Integral-Derivative) controller, and the secondary control loop uses a fractional-order PI (Proportional- Integral) controller.

$$G_{FOPID1}(s) = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu} \quad (12)$$

$$G_{FOPID2}(s) = k_p + \frac{1}{\tau_i s} \quad (13)$$

2.5.2 Controller Design of FOPCC

In parallel fractional order cascade control loop, two different fractional order controllers were employed. In parallel fractional order controller two different loops are formed. Secondary loop consists of fractional order PI controller, and primary loop consists of fractional order PID controller. (12) and (13) represent the primary and secondary controller respectively.

III. SIMULATION RESULTS

The proposed strategy's closed loop performance is contrasted with various recently reported techniques. A unit step input is simulated and compare the performance indices are compared and to show the improvement achieved by proposed scheme. We compare five parameters, ISE, IAE, ITAE, T_r, T_s.

$$\text{ISE}=\int_0^{\infty} e^2(t)\,dt \quad (14)$$

$$\text{IAE}=\int_0^{\infty} \left|e(t)^2\right| dt \quad (15)$$

$$\text{ITAE}=\int_0^{\infty} te(t)\,dt \quad (16)$$

$$T_r = \pi - Cos\frac{\xi}{\omega_d} \quad (17)$$

$$T_s = \frac{4}{\xi\omega_n} \quad (18)$$

3.1 Case-I

The output of the fractional order serries cascade control and parallel cascade control are compared. In this comparison method, the primary and secondary process are first order plus delay (FOPTD) system [10].

$$G_{P1} = G_{PM1} = \frac{e^{-4s}}{(30s+1)} \quad (19)$$

$$G_{P2} = G_{PM2} = \frac{e^{-s}}{(20s+1)} \quad (20)$$

The tuning rules derived in section-2 yields the controller setting $k_{p1} = 1.012, t_{d1} = 2.7, t_{i1} = 1388, \lambda_1 = 0.24, \mu_1 = 3$ and $k_{p2} = 1.437, t_{d2} = 2.57, t_{i2} = 1432, \lambda_2 = 0.252, \mu_1 = 3.5$ with these controller setting, the comparison of closed loop systems is compared with application of step input and unit step disturbance at t=100sec. the closed loop time response specification and error criteria as shown in table-1.

TABLE I. THE PERFORMANCE INDEX COMPARISON FOPCC AND FOSCC FOR CASE I

Method	IE	ISE	IAE	ITAE
FOPCC	21.08	32.69	62.76	4151.27
FOSCC	24.34	37.17	68.94	5737.38

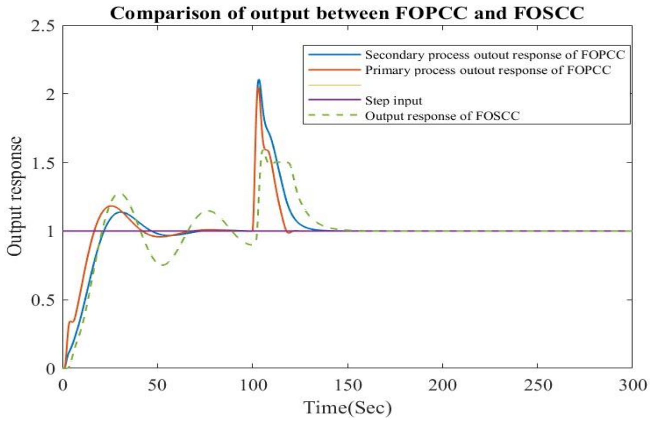

Fig. 5. Comparison of output between FOPCC and FOSCC

3.2 Case-II

Consider the primary and secondary process models that are considered $G_{p1} = G_{pd1} = \frac{0.8\,e^{-6.56}}{3.5\,s^2+s+1}$ $G_{p2} = G_{pd2} = \frac{e^{-6.56}}{1.2s+1}$. The tuning rules derived in Section-2, yields the following controller settings, $k_{p1} = 1.042, t_{d1} = 3, t_{i1} = 1682, \lambda_1 = 0.2, \mu_1 = 3$ and $k_{p2} = 1.57, t_{d2} = 2.5, t_{i2} = 1332, \lambda_2 = 0.23, \mu_1 = 3.5$

With these controller settings, performance of closed loop output response of FOSCC and FOPCC as shown in figure-6. Two processes are excited with step input and introducing negative step disturbance at t=100 sec. The time response specification and error criteria as shown in table-2.

TABLE II. THE PERFORMANCE INDEX COMPARISON FOPCC AND FOSCC FOR CASE II

Method	IE	ISE	IAE	ITAE
FOPCC	31.08	42.69	72.76	7111.27
FOSCC	34.34	42.17	78.94	7737.38

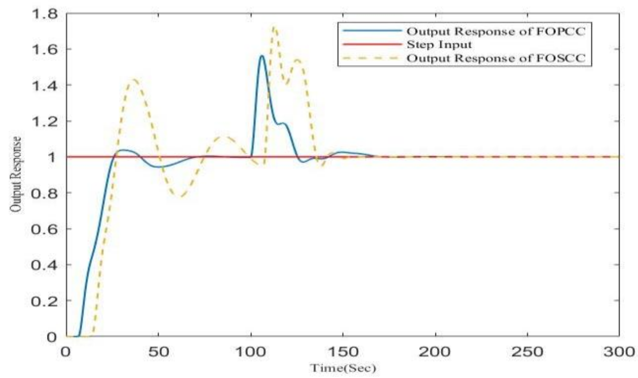

Fig. 6. Comparison of output between FOPCC and FOSCC with primary process SOPTD and secondary process FOPTD

3.3 Case-III

The output of the fractional order parallel cascade control is compared with Raja and Santosh. In this comparison method, the primary and secondary process are first order plus delay (FOPTD) system [10][11].

The tuning rules derived in section-2 yields the controller setting $k_{p1} = 1.012, t_{d1} = 2.75, t_{i1} = 1488, \lambda_1 = 0.34, \mu_1 = 3$ and $k_{p2} = 1.437, t_{d2} = 2.57, t_{i2} = 1532, \lambda_2 = 0.352, \mu_1 = 4$ with these controller setting, the comparison of closed loop output response is compared with Raja and Santosh with application of step input and unit step disturbance at t=100sec. the output response as shown in figure-7.

TABLE III. THE PERFORMANCE INDEX COMPARISON FOPCC AND FOSCC FOR CASE III

Process loop	IE	ISE	IAE	ITAE
FOPCC	9.75	14.75	24.04	1004.1
FOSCC	-2.179	20.91	38.21	2073.73

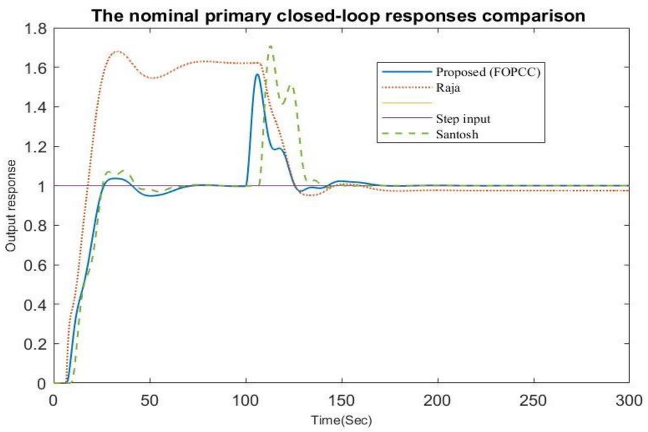

Fig. 7. Comparison of output between FOPCC with Raja[10] and Santosh[11]

IV. CONCLUSIONS

A comparison between fractional order parallel cascade control (FOPCC) and fractional order series cascade control (FOSCC) is presented in this work. This suggested approach uses two PID controllers of fractional order. An overview of the temporal response properties of the FOSCC and FOPCC is provided in this publication. The output characteristics of the parallel cascade control provide error criteria as well as a definition of the batter time response. Settling time (T_s), Rise time (T_r), Peak overshoot (T_p), Integral Error (IE), Integral Absolute Error (IAE), Integral Square Error (ISE) and Integral Time Absolute Error (ITAE) are all smaller when using the FOPCC. The suggested FOPCC is contrasted with recently published techniques. In the presence of model parameter uncertainty as well as nominal conditions, the suggested approach provides a better response.

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